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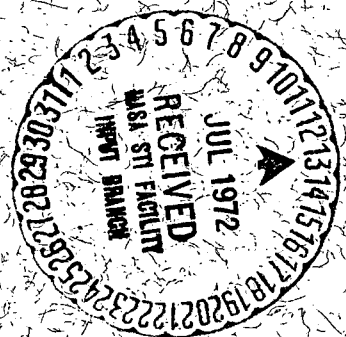
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RECOVERY OF THRUST MAGNITUDE FROM MINITRACK DATA FOR THE SERT-II SPACECRAFT

E. M. JONES

DECEMBER 1970



GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

An estimation of the thrust magnitude of the Space Electric Rocket Test (SERT) engine is obtained. This estimation is based on changes in the SERT-II spacecraft orbit as determined from tracking data by using a revised Definitive Orbit Determination System (DODS) which includes the effect of thrust upon the motion of a spacecraft in general and the SERT-II Satellite in particular. The results of these computations are found to be in good agreement with values of acceleration as determined by an on-board accelerometer. A discussion of the mathematical model used to obtain the numerical results is also presented.

RECOVERY OF THRUST MAGNITUDE FROM MINITRACK DATA FOR THE SERT-II SPACECRAFT

INTRODUCTION

The primary objective of the SERT-II mission was to provide a long term evaluation of the performance and reliability of an ion thruster system while in the space environment.

The SERT-II satellite was launched from Space Launch Complex 2 East (SLC-2E) at Vandenberg Air Force Base (VAFB) by a THORAD (SLV-2H)/Agena launch system February 4, 1970. The SERT-II satellite was placed in a nearly circular (eccentricity of 0.0008) orbit at 99.1 degrees inclination, retrograde, with a semi-major axis of 7379.5 kilometers and an anomalistic period of 105.15 minutes.

Figures 1A and 1B are the composite drawings of the SERT-II satellite. The spacecraft is a cylindrical structure approximately 0.533 meter long and 1.50 meter in diameter. It is constructed of aluminum channels, trusses, and annular beams that attach to the eight hard points on the standard shroud adapter ring of the Agena. The spacecraft is powered by solar arrays mounted on the Agena aft rack.

The ion thrusters are oriented such that when the spacecraft is in its normal attitude they are in the orbit plane with their thrust vectors passing through the spacecraft center of gravity with an offset of 9.5 degrees from the yaw axis. Under nominal attitude conditions the yaw axis is directed from the center of mass of the spacecraft and away from the earth's center; the roll axis is in the plane of the orbit perpendicular to the yaw axis; and the pitch axis is normal to the orbit plane (see Figures 8a and 8b in Appendix).

It is desirable to have the orbit geometry change significantly in order to provide a positive indication from the tracking data of the thruster performance. The location and orientation of the thrusters is important in terms of how the thrust vector may affect the orbit. Thrust applied tangential to the orbit path will produce an increase or decrease of the semi major axis of the orbit. A thrust component directed radially outward from the earth or one normal to the plane of the orbit has much smaller relative effect on the geometry of the orbit. To minimize disturbing torques the engine was always oriented so that the thrust vector passed as nearly as possible through the center of mass of the spacecraft (see reference 1).

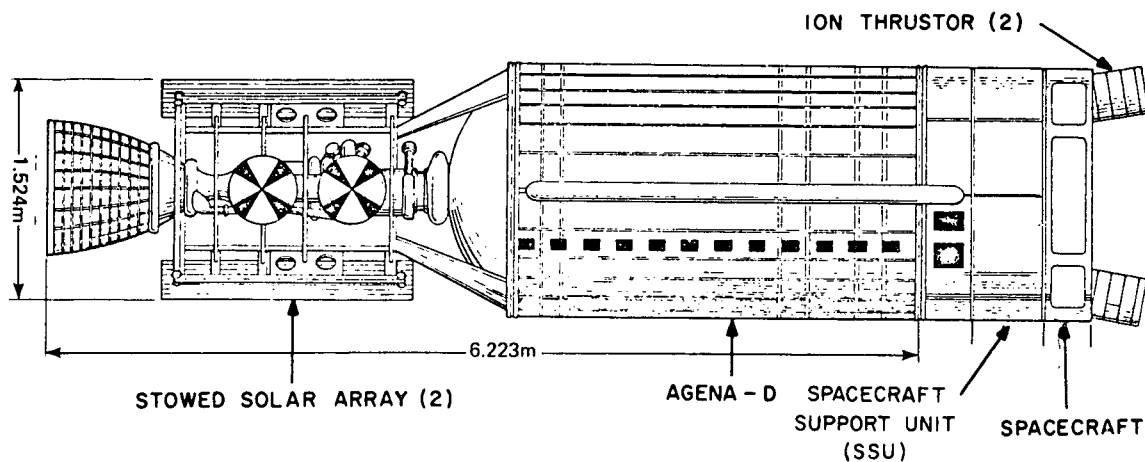


Figure 1A. SERT-II Spacecraft With Agenda-D.

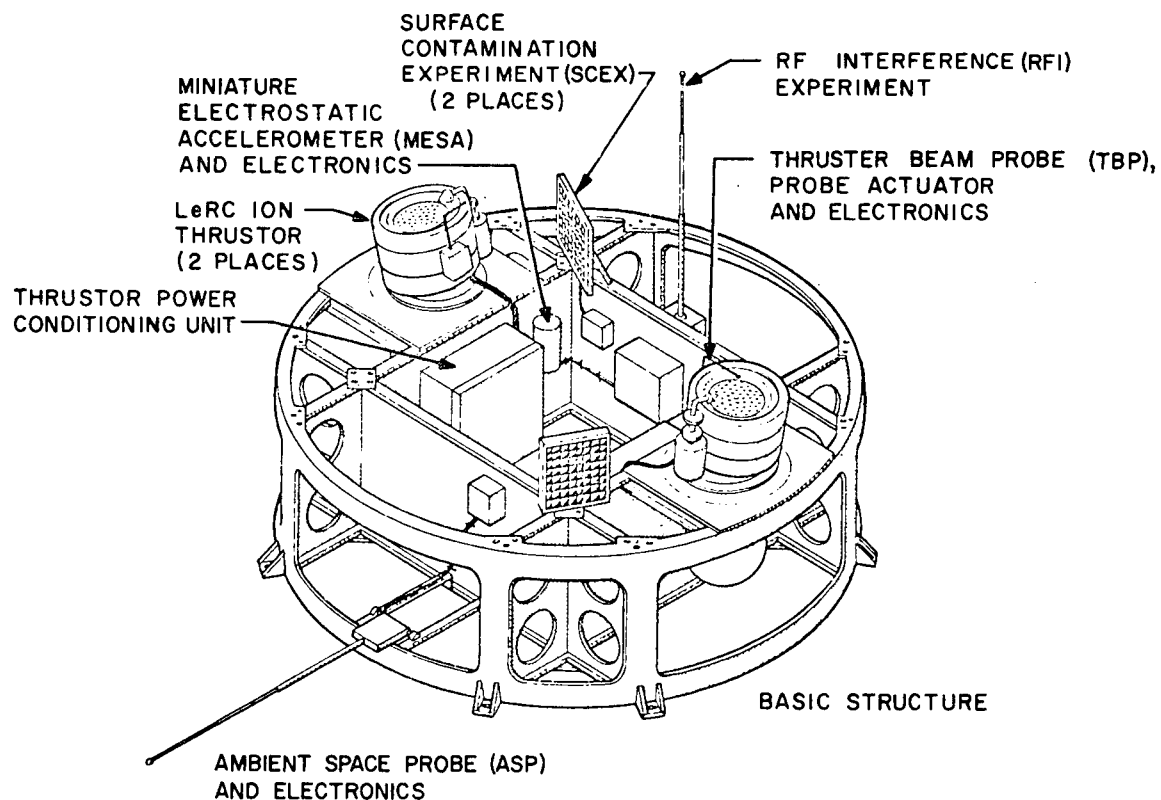


Figure 1B. SERT-II Spacecraft.

The attitude control system (ACS) is a semi-passive system used to provide three-axis stabilization. The system uses the natural gravity-gradient effect on the Agena (without booms) along with two single degree-of-freedom control moment gyros which provide damping and restoring torques.

OPERATIONS

Early orbit determinations showed that the orbit achieved was nearly the nominal orbit. Below are orbital parameters for pre-launch trajectory and actual trajectory obtained at launch.

| <u>Pre-Launch</u> <u>Epoch Feb. 4, 1970</u> | | <u>Achieved</u> <u>Epoch Feb. 4, 1970</u> <u>2 hrs 49 min 49.815 sec UT</u> |
|--|--------|---|
| Semi-Major Axis (km) | 7375.9 | 7379.5 |
| Eccentricity | .0003 | .0008 |
| Inclination (deg) | 99.1 | 99.1 |
| Height of Perigee (km) | 995.6 | 995.4 |
| Height of Apogee (km) | 999.9 | 1007.3 |
| Anomalistic Period (min) | 105.07 | 105.13 |

The first week of the SERT-II mission was free of any thrusting maneuvers. A planned schedule of events, consisting of varying the thrust levels of the ion engine, was performed from Feb. 11, 1970 to Feb. 14, 1970. Through this time interval the number one thruster was operated as the principal thruster and thruster number two was held in readiness as the backup thruster. Three levels of thrust magnitude were tested. They were 30%, 80% and 100% of full thrust. Either thruster could be oriented so as to increase or decrease the semi-major axis. During this test, thruster number one was oriented so as to increase the semi-major axis. Table 1 shows each event involving thruster manipulation.

Upon completion of tests with the ion engine at different thrust levels, a measurement by the on-board accelerometer gave the thrust to be 2.82×10^3 dynes. (According to a telecon with Lewis Research Center (LeRC) the accelerometer was calibrated to $\pm 4\%$ accuracy for pre-flight tests.)

Orbit determination based on Minitrack data from the STADAN network and attitude data supplied to Goddard Space Flight Center (GSFC) by Lewis Research Center (LeRC) gave 0.845×10^3 dynes as a first estimate of the thrust at the 30% level, which when compared with the measurement taken by the on-board

Table 1

| Date | | | | | Thruster Number | Thrust Level Percent |
|------|-------|-----|------|--------|--------------------|-------------------------|
| Year | Month | Day | Hour | Min | | |
| 70 | 02 | 11 | 01 | 10 EST | 2 | 30 |
| 70 | 02 | 11 | 18 | 51 EST | 2 | 80 |
| 70 | 02 | 12 | 03 | 24 EST | 2 | 100 |
| 70 | 02 | 13 | 03 | 54 EST | 2 | 80 |
| 70 | 02 | 13 | 07 | 05 EST | 2 | 30 |
| 70 | 02 | 13 | 09 | 00 EST | 2 | None (Off) |
| 70 | 02 | 13 | 23 | 23 EST | 1 | None (Preheat) |
| 70 | 02 | 14 | 02 | 51 EST | 1 | 30 |
| 70 | 02 | 14 | 16 | 06 EST | 1 | 100 |

accelerometer differed by one part in a hundred. Later orbit arcs gave the thrust magnitude at the 100% (i.e., 2.82×10^3 dynes) thrust level within ± 2 parts in a hundred when compared to the accelerometer value.

Determining thrust magnitude from tracking data, coupled with attitude data, has shown that if we take the one time accelerometer measurement of 2.82×10^3 dynes as the norm, the SERT-II orbit determination system's capability for recovering thrust magnitude displays a remarkable accuracy - 99 percent of the value as measured by the accelerometer. This is well within the accuracy of the accelerometer calibration. It was originally anticipated that recovery of thrust magnitude from tracking data might only agree to within 80% of the thrust measured by the accelerometer.

On February 23, 1970 trouble developed in the accelerometer. Several attempts were made to correct the malfunction with no success. With the accelerometer no longer functioning, the thrust magnitude was now solely determined from tracking data.

The SERT-II spacecraft operated with thruster system number one for 3,782 hours. This hour count was not continuous since thruster system number one was shut down twice. On March 7, 1970 the ion engine was shutdown during the solar eclipse to preserve power and on May 21, 1970 the voltage regulator received an overcharge causing automatic shutdown of the ion engine. On each occasion of shutdown the restart procedure for orbit determination presented no special problems. The SERT-II Orbit Determination System using Minitrack data was used to recover thrust magnitude for the 100% (i.e., 2.82×10^3 dynes) level of thruster system number one throughout the 3,782 hours of operation, which ended July 20, 1970.

Following the shutdown of the ion engine on July 20, 1970, several attempts were made to restart thruster system number one, with no success. The conclusion of LeRC was that a metallic short existed between the accelerator and the screen grids, causing failure of the thruster.

On July 24, 1970, 16 hours 15 minutes universal time, thruster number two was activated. The orientation of thruster number two was such that it should cause a decrease in the semi-major axis. Analysis of the orbit determination restart procedure showed that the SERT satellite was now accelerating back toward earth. The differential correction process indicated this reversal of acceleration by computation of a negative thrust with a magnitude slightly less than half the 100% value of 2.82×10^3 dynes, i.e.: the thrust magnitude recovered from tracking data now yielded a value -1.41×10^3 dynes.

It had been anticipated that a change of thrusters would only reverse the direction of the thrust so that altitude would now decrease but that the thrust magnitude would remain 2.82×10^3 dynes. From results based on Minitrack data for Thruster system number two it appears that the thruster was operating at approximately one-half of its efficiency. This was probably due to incorrect alignment causing part of the thrust to be dissipated out of the plane of the orbit (i.e., causing a minute change in inclination).

Thruster number two was shutdown on Aug. 31, 1970 to again preserve power during a solar eclipse. On Oct. 17, 1970 thruster number two was shutdown because of high current buildup. As a result of thruster system two shutting down, the SERT-II spacecraft is now in a passive orbit with regards to thrusting capabilities.

Before shutdown of thruster system two, it operated for approximately 2,011 hours. Thus the total time of both thruster systems was 5,793 hours of actual thrusting.

Examinations of both the free flight and thrusting portion of SERT-II orbits with a differential correction process shows the residuals from a least squares process to be of exceptional quality. The average root mean square of the direction cosines for all orbit arcs is as follows:

| <u>Free Flight</u> | <u>Thruster-On</u> |
|--|--|
| $\ell = 0.1814 \times 10^{-3}$ radian $\approx 35''$ | $\bar{\ell} = 0.2274 \times 10^{-3}$ radian $\approx 45''$ |
| $m = 0.2382 \times 10^{-3}$ radian $\approx 49''$ | $\bar{m} = 0.2479 \times 10^{-3}$ radian $\approx 50''$ |

Figure 2 depicts the quality of the tracking data used in determining thrust magnitude. Examination of the frequency histograms reveals most of the residuals from a least squares process fall within the range of 1×10^{-4} to 4×10^{-4} radian.

A table of thrust magnitude at the 100% thrust level for thruster number one and two (Table 2) and a graph of thrust magnitude (Figure 3) versus time in days shows the results of thrust magnitude recovered from tracking data. A comparison of the one time accelerometer (i.e., 2.82×10^3 dynes) with the plotted values of Figure 3 shows the differential correction process performed admirably in recovering thrust magnitude from tracking data.

Computation of the mean thrust was obtained using the equation:

$$|\bar{P}_{mj}| = \sum_{i=1}^n |\bar{P}_i| / N$$

where

$|\bar{P}_i|$ = thrust magnitude from a differential correction process

N = total number of orbit arcs

The means computed for thruster number one and two are:

$$|\bar{P}_{m1}| = 2.81 \times 10^3 \text{ dynes}$$

$$|\bar{P}_{m2}| = -1.43 \times 10^3 \text{ dynes}$$

The differential correction to one or more of the thrust parameters required the Definitive Orbit Determination System (DODS) to be altered in a manner that solves the basic problem of providing partial derivatives of the tracking observations with respect to the unknown thrust parameters. A new package was developed to add thrust to the force module already existing in the DODS orbit system. See Appendix A for description of mathematical model used in the SERT-II Orbit Determination System.

CONCLUSION

Comparison of the on-board accelerometer measurement (2.82×10^3 dynes), with the results from a differential correction process (2.81×10^3 dynes), and the nominal value (2.76×10^3 dynes) shows conclusively that thrust magnitude could be recovered satisfactorily using tracking data.

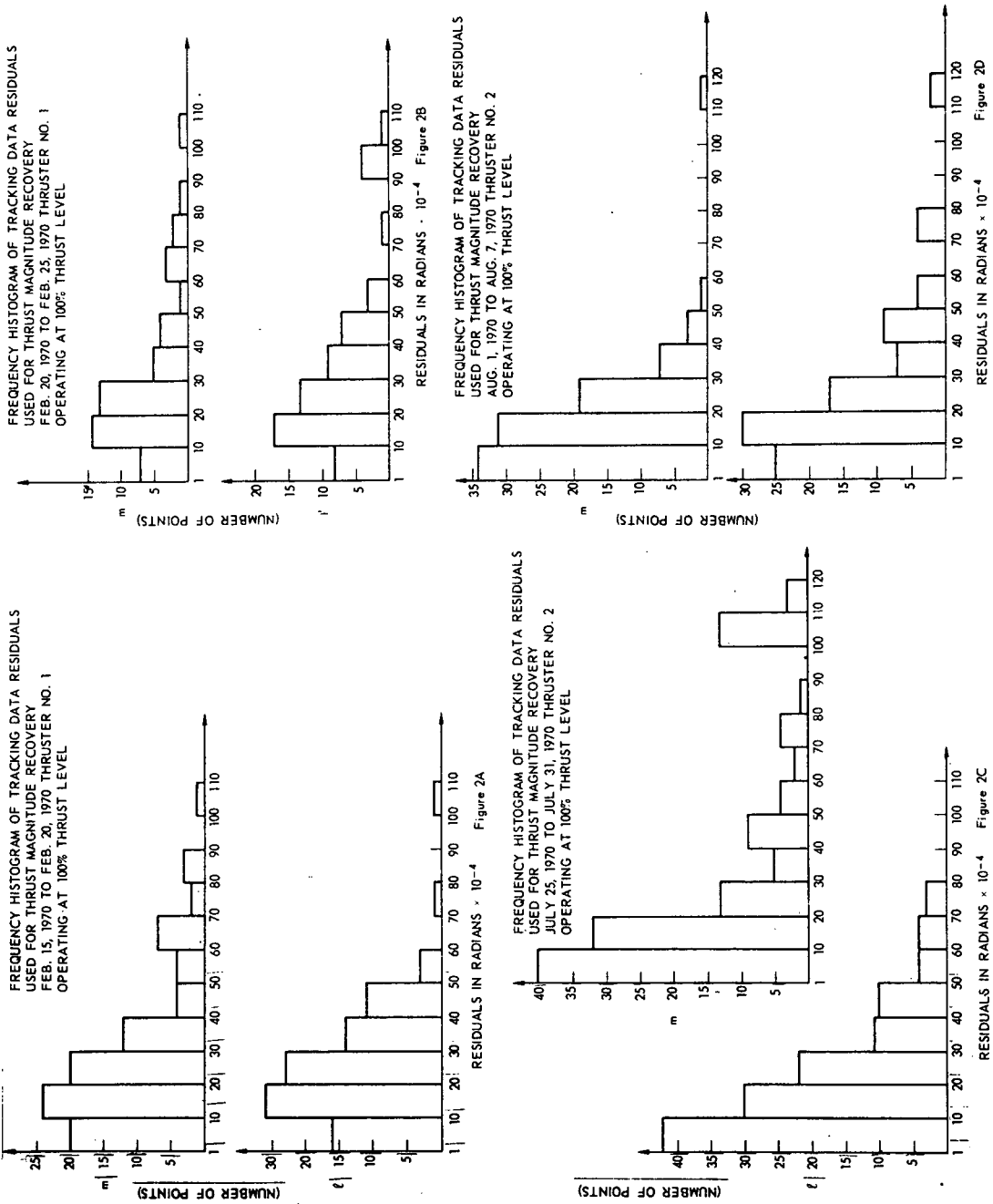


Figure 2. Frequency Histogram for Tracking Data Distribution.

Table 2
Thrust Magnitude of Orbit Arcs from a Differential Corrector Process

| Orbit Determination Period | | | | | | Thrust Magnitude | |
|----------------------------|----|----|----------------|----|----|----------------------------------|--------|
| Year/Month/Day | | | Year/Month/Day | | | Millipounds, Dynes $\times 10^3$ | |
| 70 | 02 | 16 | 70 | 02 | 20 | 6.363 | 2.830 |
| 70 | 02 | 20 | 70 | 02 | 24 | 6.353 | 2.826 |
| 70 | 02 | 24 | 70 | 02 | 27 | 6.340 | 2.820 |
| 70 | 02 | 27 | 70 | 03 | 01 | 6.342 | 2.821 |
| 70 | 03 | 02 | 70 | 03 | 06 | 6.352 | 2.825 |
| 70 | 03 | 05 | 70 | 03 | 07 | 6.334 | 2.817 |
| 70 | 03 | 08 | 70 | 03 | 13 | 6.331 | 2.816 |
| 70 | 03 | 12 | 70 | 03 | 17 | 6.330 | 2.816 |
| 70 | 03 | 16 | 70 | 03 | 21 | 6.326 | 2.814 |
| 70 | 03 | 20 | 70 | 03 | 25 | 6.317 | 2.810 |
| 70 | 03 | 24 | 70 | 03 | 30 | 6.313 | 2.808 |
| 70 | 03 | 29 | 70 | 04 | 04 | 6.299 | 2.802 |
| 70 | 04 | 03 | 70 | 04 | 10 | 6.296 | 2.800 |
| 70 | 04 | 09 | 70 | 04 | 15 | 6.296 | 2.800 |
| 70 | 04 | 14 | 70 | 04 | 20 | 6.293 | 2.799 |
| 70 | 04 | 19 | 70 | 04 | 25 | 6.291 | 2.798 |
| 70 | 04 | 24 | 70 | 04 | 30 | 6.288 | 2.797 |
| 70 | 04 | 29 | 70 | 05 | 05 | 6.284 | 2.795 |
| 70 | 05 | 04 | 70 | 05 | 10 | 6.291 | 2.798 |
| 70 | 05 | 09 | 70 | 05 | 15 | 6.292 | 2.799 |
| 70 | 05 | 14 | 70 | 05 | 20 | 6.302 | 2.803 |
| 70 | 05 | 19 | 70 | 05 | 21 | 6.312 | 2.808 |
| 70 | 05 | 22 | 70 | 05 | 27 | 6.297 | 2.801 |
| 70 | 05 | 26 | 70 | 06 | 01 | 6.284 | 2.795 |
| 70 | 05 | 31 | 70 | 06 | 06 | 6.295 | 2.800 |
| 70 | 06 | 05 | 70 | 06 | 11 | 6.286 | 2.796 |
| 70 | 06 | 10 | 70 | 06 | 16 | 6.281 | 2.794 |
| 70 | 06 | 15 | 70 | 06 | 21 | 6.293 | 2.799 |
| 70 | 06 | 20 | 70 | 06 | 26 | 6.302 | 2.803 |
| 70 | 06 | 25 | 70 | 07 | 01 | 6.307 | 2.805 |
| 70 | 06 | 30 | 70 | 07 | 06 | 6.309 | 2.806 |
| 70 | 07 | 05 | 70 | 07 | 11 | 6.305 | 2.804 |
| 70 | 07 | 10 | 70 | 07 | 16 | 6.303 | 2.804 |
| 70 | 07 | 15 | 70 | 07 | 21 | 6.294 | 2.799 |
| 70 | 07 | 20 | 70 | 07 | 23 | 6.277 | 2.792 |
| 70 | 07 | 25 | 70 | 07 | 31 | -4.169 | -1.854 |
| 70 | 08 | 01 | 70 | 08 | 07 | -3.169 | -1.410 |
| 70 | 08 | 06 | 70 | 08 | 12 | -3.173 | -1.411 |
| 70 | 08 | 11 | 70 | 08 | 17 | -3.175 | -1.412 |
| 70 | 08 | 16 | 70 | 08 | 22 | -3.176 | -1.413 |
| 70 | 08 | 21 | 70 | 08 | 27 | -3.177 | -1.413 |
| 70 | 08 | 26 | 70 | 09 | 01 | -3.182 | -1.415 |
| 70 | 09 | 04 | 70 | 09 | 10 | -3.189 | -1.418 |
| 70 | 09 | 09 | 70 | 09 | 15 | -3.191 | -1.419 |
| 70 | 09 | 14 | 70 | 09 | 20 | -3.194 | -1.421 |
| 70 | 09 | 19 | 70 | 09 | 25 | -3.203 | -1.425 |
| 70 | 09 | 24 | 70 | 09 | 30 | -3.207 | -1.426 |
| 70 | 09 | 29 | 70 | 10 | 05 | -3.213 | -1.429 |
| 70 | 10 | 04 | 70 | 10 | 10 | -3.224 | -1.434 |
| 70 | 10 | 09 | 70 | 10 | 15 | -3.237 | -1.440 |
| 70 | 10 | 14 | 70 | 10 | 18 | -3.252 | -1.446 |

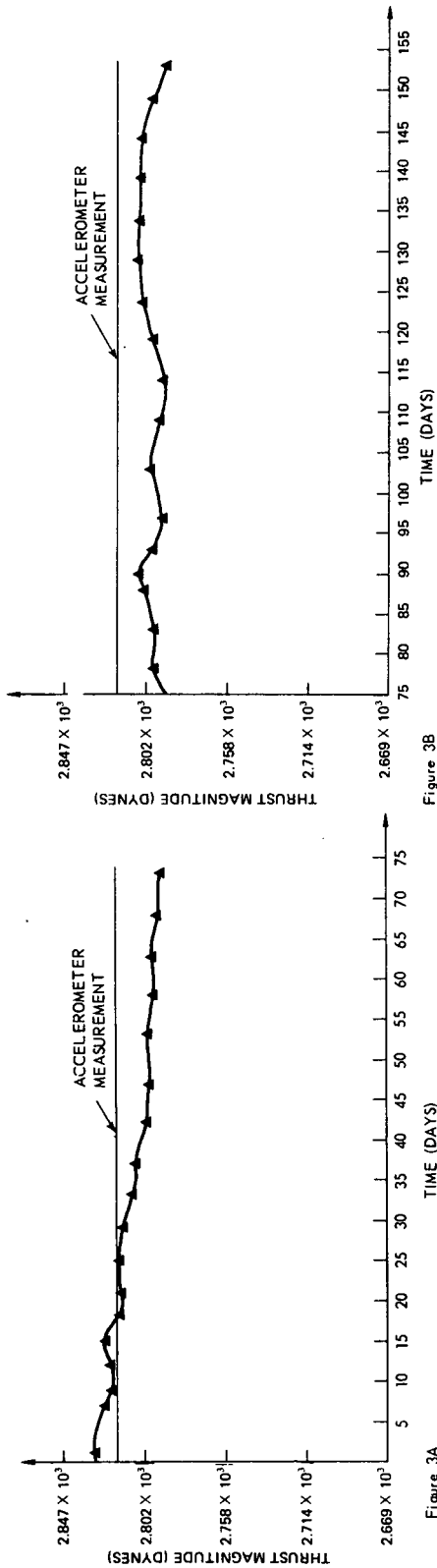


Figure 3B

THRUST MAGNITUDE 100% LEVEL THRUSTER NO. 1

Figure 3C

THRUST MAGNITUDE 100% LEVEL THRUSTER NO. 2

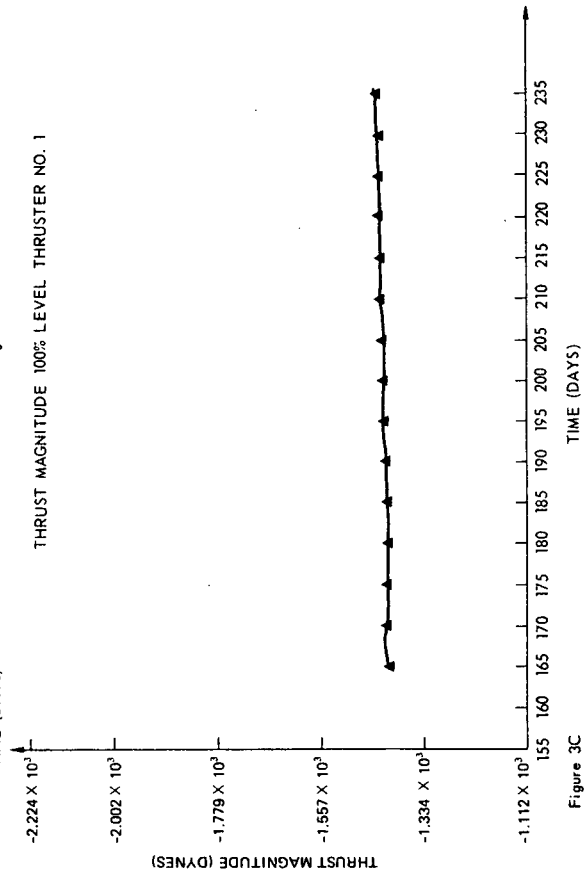


Figure 3. Thrust Performance at Full on for Thruster Number 1 and 2.

Orbit computations performed while thruster system number one was operating during the period of Feb. 14, 1970 to July 20, 1970 indicated the thruster operated at approximately 99.5% of the accelerometer measurement. Thruster system two operating during the period of July 24, 1970 to Aug. 31, 1970 had an efficiency of approximately 50.8% of the value it was anticipated that it would have.

The SERT-II mission has demonstrated that an ion engine in a space environment can perform over an extended period of time. The explained failure of thruster number two to perform at the 100% level does not negate this conclusion.

Acknowledgments

The author gratefully acknowledges valued discussions with Dr. David Blanchard, Mr. Werner D. Kahn and Mr. David J. Stewart of Trajectory and Dynamics Branch, Trajectory Analysis and Geodynamics Division.

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2. "NASA Support Plan," Goddard Space Flight Center, July 26, 1968.
3. SERT Spacecraft, "Thrust and Orbit Determination Program," Definitive Orbit Determination System - Model 2, Computer Science Corporation, June 2, 1969.

APPENDIX

DEVELOPMENT

An examination of the mathematical development involving thrust magnitude, attitude angles, and gimbal angles suggests that the most practical approach is to represent these parameters as second degree polynomials with respect to time and reduce them to quantities expressed in an inertial coordinate frame.

Hence $|\bar{P}|$ could be expressed as:

$$|P| = P_1 + P_2 t + P_3 t^2$$

where $|P|$ equals thrust magnitude and P_1, P_2, P_3 are the parameters to be determined in a least squares sense.

The force module in the Definitive Orbit Determination System (DODS) was extended to include the thrust acceleration.

Thus

$$\ddot{X} = \ddot{X}_{TWO} + \ddot{X}_G + \ddot{X}_D + \ddot{X}_{SR} + \ddot{X}_{SG} + \ddot{X}_{LG} + \ddot{X}_T$$

where

\ddot{X}_{TWO} = acceleration of two body term

\ddot{X}_G = acceleration from geopotential

\ddot{X}_D = acceleration from drag

\ddot{X}_{SR} = acceleration from solar radiation

\ddot{X}_{SG} = acceleration from solar gravity

\ddot{X}_{LG} = acceleration from lunar gravity

\ddot{X}_T = acceleration from thrust

The basic coordinate system used in the Definitive Orbit Determination System (DODS) to generate a trajectory is a celestial cartesian system. The thrust acceleration vector was integrated in the form

$$M = M_0 + \int_{t_0}^t \dot{m} dt$$

where

$$\bar{a} = -\frac{\dot{m} \bar{\omega}_e}{M} = \frac{\bar{P}}{M} \quad \text{or} \quad \bar{P} = \dot{m} \bar{\omega}_e$$

\dot{m} = mass fuel flow rate

M = the instantaneous mass of vehicle

M_0 = the mass of vehicle at epoch

\bar{P} = thrust vector

$\bar{\omega}_e$ = effective gas velocity

Thus from nominal thrust magnitude obtained from telemetry data the instantaneous mass fuel flow was computed as:

$$\dot{m} = \frac{|\bar{P}|}{\text{ISP} \cdot g}$$

where g is the gravitational acceleration at sea level and ISP is the specific impulse of the propulsion system.

In the SERT-II computation system the initial mass of the vehicle and the specific impulse are inputs. The orientation of the thrust vector and its magnitude were obtained from telemetry attitude, nozzle angles, and thrust magnitude data.

The differential correction to one or more of the thrust parameters required DODS to be altered in a manner that solves the basic problem of providing partial derivatives of the tracking observations with respect to the unknown thrust parameter for the least squares process.

It will be shown in the subsequent development that the thrust defined above is related to the acceleration ($\ddot{\mathbf{X}}_T$) by

$$\ddot{\mathbf{X}}_T = \frac{\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}}{M}$$

Since the satellite position vectors \vec{r} and \vec{r} are given in the celestial coordinate system, while the thrust occurs in a body fixed frame, it is convenient to define an instantaneous flight plane system (U, V, W) in which the unit vectors are given by,

$$\hat{U} = \frac{\vec{r}}{|\vec{r}|}, \quad \hat{V} = \frac{\vec{r} \times \vec{r}}{|\vec{r} \times \vec{r}|}, \quad \hat{W} = \hat{U} \times \hat{V}.$$

The center of the UVW system is at the center of mass of the satellite (Figure 4). \hat{U} is directed along the geocentric radius vector \vec{r} , \hat{V} is perpendicular to the flight plane containing \vec{r} and \vec{r} , and W is a unit vector completing the right-handed system. The UVW system is related to the thrust vector through the thrust variables (thrust magnitude, gimbal angles, and attitude angles). It is also related to the geocentric celestial coordinate system through \vec{r} and \vec{r} . Thus our desired celestial acceleration due to the thrust will be computed through the UVW system.

Let X_b, Y_b, Z_b be a local spacecraft (body-fixed) cartesian coordinate system with center at the center of mass of the spacecraft (Figure 5). Let Y_b , the roll axis point positive in the direction of the front of the vehicle, let Z_b , the yaw axis, point up through the top of the vehicle and X_b , the pitch axis, complete a right handed system. (The thrust vector is in the $Y_b - Z_b$ plane.)

The X_b, Y_b, Z_b system is related to the UVW system by a set of ordered rotations, the attitude angles (pitch, yaw, and roll). This relationship can be obtained by initially letting X_b^o, Y_b^o, Z_b^o be coincident with $\hat{V}, \hat{W}, \hat{U}$ respectively. Then making a yaw rotation about Z_b^o through the negative angle τ followed by a pitch rotation about X_b^o through the angle α will produce the axis X_b', Y_b', Z_b' , and finally a roll rotation about the Y_b' axis through the angle θ will relate the X_b, Y_b, Z_b axes to the UVW system. The F matrix shows these rotations.

The thrust vector is usually given in the form of thrust magnitude and gimbal angles with respect to the X_b, Y_b, Z_b coordinates.

The components of thrust in the X_b, Y_b, Z_b direction are given by

$$\begin{pmatrix} P_{x_b} \\ P_{y_b} \\ P_{z_b} \end{pmatrix} = G |P|$$

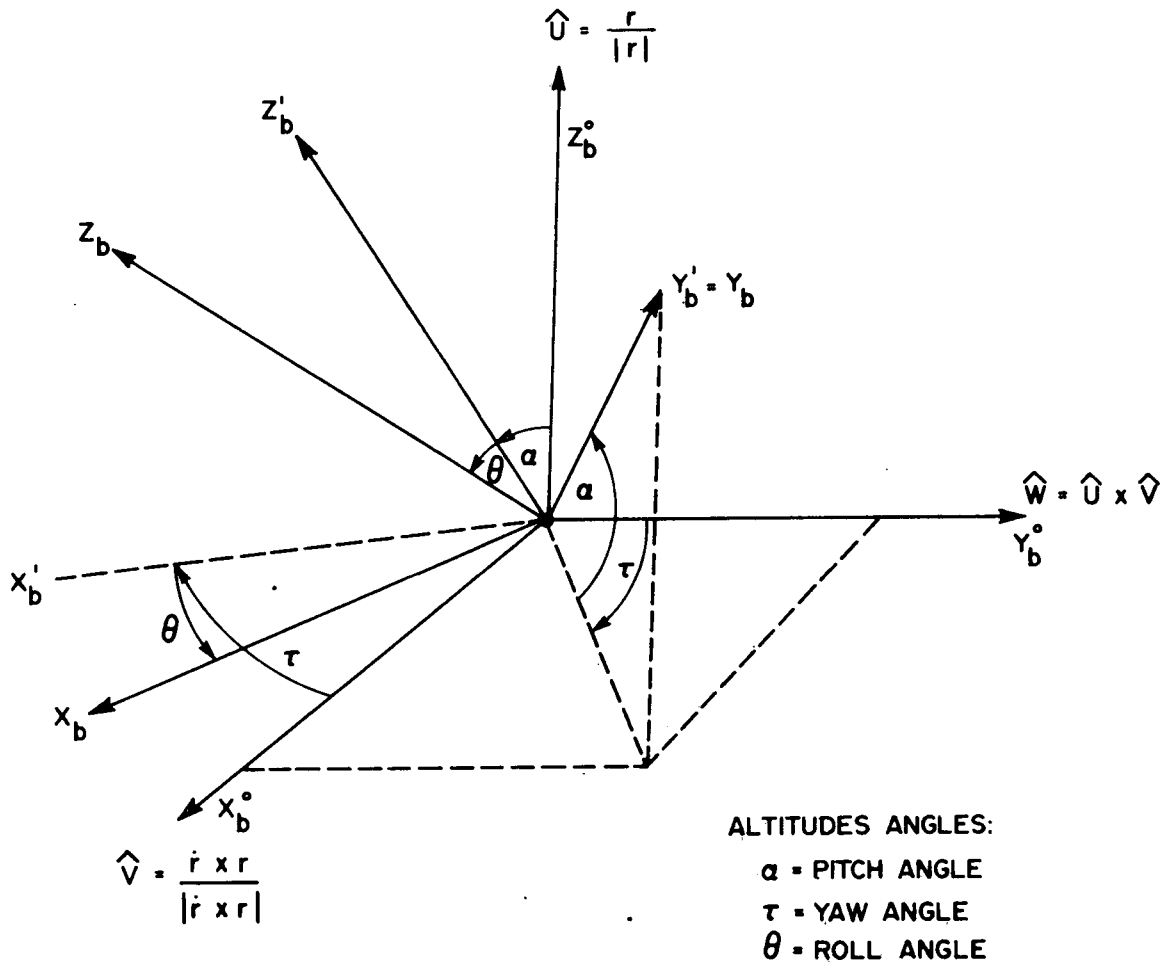


Figure 4

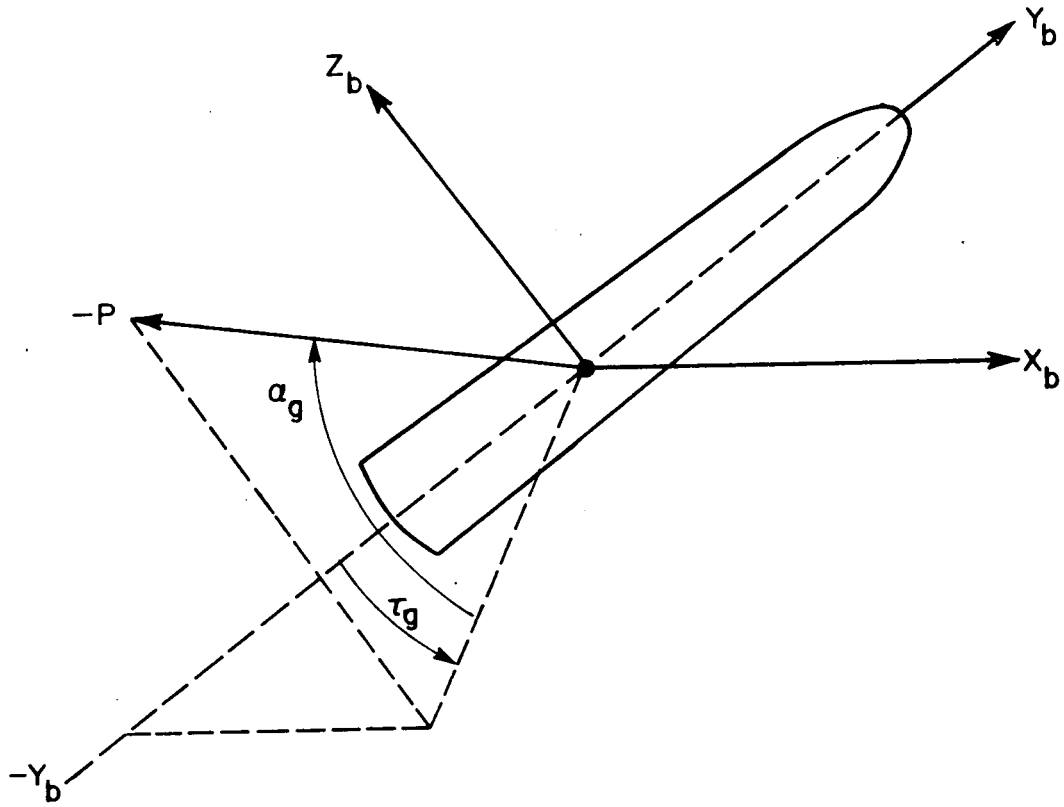


Figure 5

where

$$G = \begin{pmatrix} \cos \alpha_g \sin \tau_g \\ -\cos \alpha_g \cos \tau_g \\ \sin \alpha_g \end{pmatrix}$$

The thrust components in the UVW system are:

$$\begin{pmatrix} P_v \\ P_w \\ P_u \end{pmatrix} = F \begin{pmatrix} P_{x_b} \\ P_{y_b} \\ P_{z_b} \end{pmatrix}$$

where

$$\begin{pmatrix} X_b \\ Y_b \\ Z_b \end{pmatrix} = CBA \begin{pmatrix} V \\ W \\ U \end{pmatrix}$$

$$\begin{pmatrix} V \\ W \\ U \end{pmatrix} = A^T B^T C^T \begin{pmatrix} X_b \\ Y_b \\ Z_b \end{pmatrix}$$

$$F = \begin{pmatrix} \cos \theta \cos \tau + \sin \theta \sin \tau \sin \alpha & \sin \tau \cos \alpha & \sin \theta \cos \tau - \cos \theta \sin \tau \sin \alpha \\ -\sin \tau \cos \theta + \cos \tau \sin \theta \sin \alpha & \cos \alpha \cos \tau & -\sin \theta \sin \tau - \cos \theta \cos \tau \sin \alpha \\ -\sin \theta \cos \alpha & \sin \alpha & \cos \alpha \cos \theta \end{pmatrix}$$

In order to transform to the inertial X, Y, Z system, it is convenient to represent the thrust components in the vehicle centered ENU_p (East, North, U_p). The ENU_p system is related to the UVW system through the azimuth angle A_z . The components of thrust in the ENU_p system is given by

$$\begin{pmatrix} P_E \\ P_N \\ P_{U_p} \end{pmatrix} = D \begin{pmatrix} P_V \\ P_W \\ P_U \end{pmatrix}$$

where

$$D = \begin{pmatrix} \cos A_z & \sin A_z & 0 \\ -\sin A_z & \cos A_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

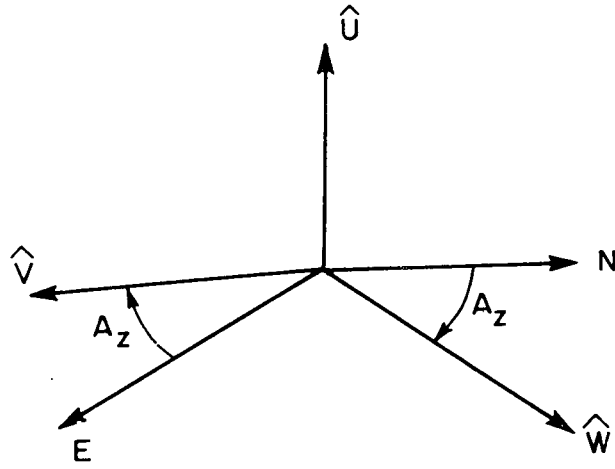


Figure 6

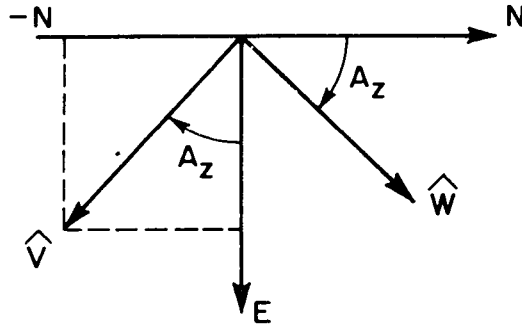


Figure 7

From Figures 6 and 7,

$$A_z = \tan^{-1} \frac{(-\hat{V} \cdot \mathbf{N})}{(\hat{V} \cdot \mathbf{E})}$$

Hence the transformation between the inertial XYZ and the ENU_p system is given by rotations in right ascension (RA) and declination (ϕ).

$$\begin{pmatrix} P_X \\ P_Y \\ P_Z \end{pmatrix} = B \begin{pmatrix} P_E \\ P_N \\ P_{U_P} \end{pmatrix}$$

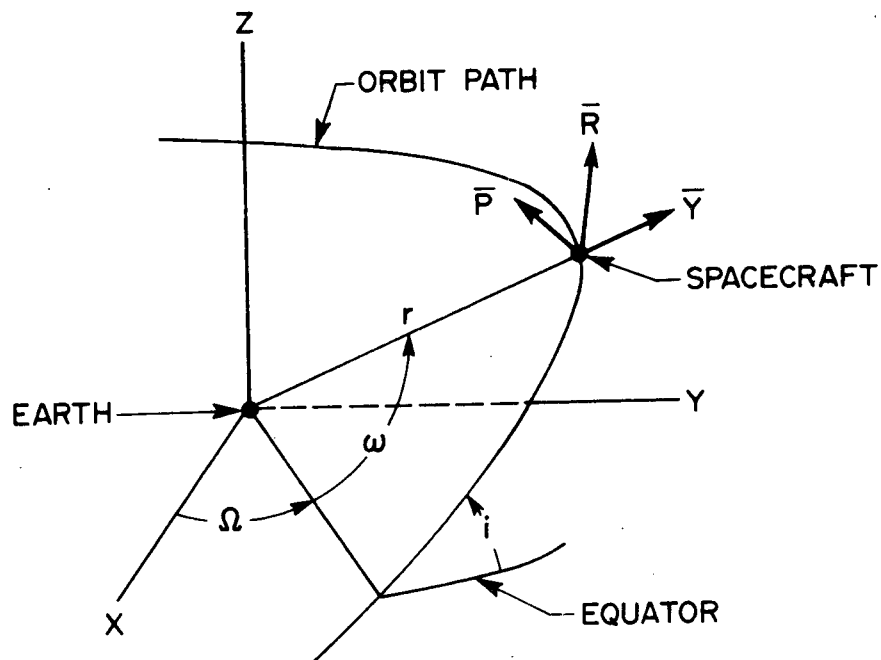
where

$$B = \begin{pmatrix} -\sin R_A & -\cos R_A \sin \phi & \cos R_A \cos \phi \\ \cos R_A & -\sin R_A \sin \phi & \sin R_A \cos \phi \\ 0 & \cos \phi & \sin \phi \end{pmatrix}$$

In summary

$$\begin{pmatrix} P_X \\ P_Y \\ P_Z \end{pmatrix} = BDFG |P|$$

$$\text{Hence } \ddot{X}_T = BDFG \frac{|P|}{M}.$$



X,Y,Z EARTH CENTERED COORDINATES

\bar{P} PITCH AXIS - NORMAL TO THE ORBIT PLANE

\bar{Y} YAW AXIS - IN ORBIT PLANE DIRECTED FROM CENTER OF EARTH

\bar{R} ROLL AXIS - PERPENDICULAR TO Y IN THE ORBIT PLANE

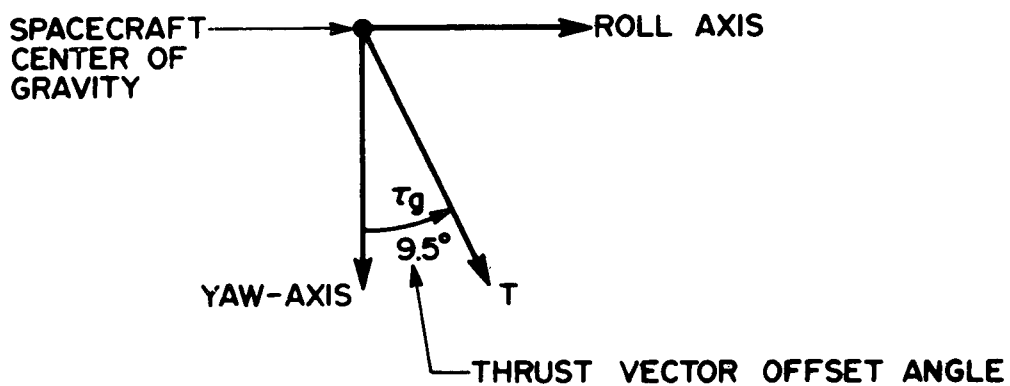
Ω LONGITUDE OF ASCENDING NODE

ω ARGUMENT OF PERIGEE

i INCLINATION

r RADIUS VECTOR

Figure 8a



PITCH AXIS NORMAL TO YAW-ROLL PLANE.

Figure 8b